LOCATION STATISTICS OF AM BROADCAST GROUNDWAVE SIGNAL AMPLITUDES

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Abstract - **A location-dependent statistical distribution model for AM broadcast groundwave signal amplitudes is presented. The distribution is in the form of a probability density function (pdf) whose expected value is the predicted field strength based on propagation formulas, and whose variance can be empirically derived from analysis of actual groundwave field strength measurements. The results are useful for assessing the percentage of locations at which a selected signal strength will be exceeded given the predicted signal strength and typical variance of the pdf.**

1.0 INTRODUCTION

For allocation purposes, the amplitude of daytime groundwave signals from AM broadcast stations has traditionally been regarded as a constant and predictable quantity when sufficiently detailed information about ground conductivity and permittivity along the propagation path is available. This view has been reinforced by the fact that in general groundwave signal amplitudes during daytime don't exhibit the time-dependent variations which are observed for AM nighttime skywave signals, or for the entire class of spacewave signals at HF and higher frequencies.

While groundwave signals do not usually exhibit time variability, the actual measured signal strength at a particular location can vary from its predicted value due to several factors:

1) errant signal contributions from reradiating manmade structures (buildings, towers, wires),
2) natural formations such as hills and abrupt conductivity boundaries which cannot be simply characterized in terms of alternating zones of conductivity and permittivity,
3) inaccurately identified conductivities and permittivities along the propagation path.

This location variability represents a source of error in predicting field strength; that is, given currently employed propagation models for predicting groundwave signal strength, the actual signal strength at a particular location will differ from the predicted value by some stochastic error function which characterizes the location variability. The degree of variability or uncertainty is proportional to the variance of the error function. In the analysis to follow, the contributing factors to the error function are represented by a gaussian-distributed random voltage which, when added to the predicted or expected voltage, produces a pdf for the sum which is a Rice-distributed random variable (r.v.) with a mean value essentially equal the predicted field strength value.

A prediction or estimation technique for the field strength can be based on the equivalent distance method described in the FCC Rules, or be based on a technique for analyzing field strength measurements which is described in a companion paper [1]. Rather than relying on the M3 map conductivity data, the latter method seeks groundwave propagation curve sections which most closely fit groups of measurement point data. The statistical information about the distribution of points around the "best fit" curve sections which represent the predicted field strengths is also naturally incorporated into the computer program described in [1]. The balance of this paper lays out specific equations which describe the pdf for signal strengths and how the variance of the pdf may be derived from the measurement data. Some preliminary results based on analysis of real data are also presented. These initial results suggest that the variance may be a complicated function which is dependent on the predicted signal strength at the prediction location, the distance of the location from the transmitter, and the degree of manmade development in the vicinity of location.

2.0 MODEL ERROR DISTRIBUTION FOR FIELD STRENGTHS

At a particular location, the envelope of the AM signal can be viewed as the sum of the predicted signal plus the composite of a number of errant signals representing the contributions of unknown reradiation sources or inaccurately specified propagation factors listed in Section 1. As a first approximation, the amplitude of the sum of the errant signals can be modelled at a gaussian r.v. (with location, not time, as the stochastic dependence), and with phase which is uniformly distributed. The sum of the predicted voltage level $A(l)$, and error voltage level $e(l)$, gives the total signal $z(l)$, at a particular location:

$$z(l) = A(l) + e(l) \tag{1}$$

If $e(l)$ is a gaussian-distributed voltage with uniform-distributed random phase, at any particular location it can be resolved into orthogonal gaussian r.v.'s, $x(l)$ and $y(l)$. Then, with $A(l)$ constant (that is, the expected value based on propagation formulas) and at zero phase, $z(l)$ becomes

$$z(l) = (x(l) + A(l))\cos\omega t + y(l)\sin\omega t \tag{2}$$

$$z(l) = r(l)\cos(\omega t + \theta) \tag{3}$$

Where r is the amplitude of the envelope at location l

$$r^2 = (x + A)^2 + y^2 \tag{4}$$

The phase of the signal θ is arbitrary and has no effect on the results for the amplitude of the envelope.

Letting $u = x + A$, we have

$$r^2 = u^2 + y^2$$

$$\theta = \tan^{-1}\left(\frac{y}{x + A}\right) = \tan^{-1}\frac{y}{u}$$

Since x and y are independent, u and y (where $u = x + A$ is gaussian with mean A) are also independent, so that

$$p(u, y) = p(u)\,p(y) \tag{5}$$

$$p(u, y) = \frac{1}{2\pi\sigma^2}\exp\left(\frac{-((u - A)^2 + y^2)}{2\sigma^2}\right) \tag{6}$$

Where σ^2 is the variance of the error signal amplitudes, x or y. Converting to polar coordinates, and noting that $du\,dy = r\,dr\,d\theta$, the joint distribution becomes

$$p(r, \theta) = p(u, y)$$

$$p(r, \theta) = \frac{1}{2\pi\sigma^2}\exp\left(\frac{-(r^2 + A^2 - 2rA\cos\theta)}{2\sigma^2}\right)$$

$$p(r, \theta)\,dr\,d\theta = \frac{r}{2\pi\sigma^2}\exp\left(\frac{-(r^2 + A^2 - 2rA\cos\theta)}{2\sigma^2}\right)dr\,d\theta$$

The distribution or pdf for the envelope only is found by integrating out the θ dependence

$$p(r) = \int_0^{2\pi} p(r, \theta)\,dr\,d\theta = \frac{r}{\sigma^2}\exp\left(\frac{-(A^2 + r^2)}{2\sigma^2}\right) I_0\left(\frac{rA}{\sigma^2}\right) \tag{7}$$

Where $I_0(z)$ is the modified Bessel function of the first kind, zero order

$$I_0(z) = \frac{1}{2\pi}\int_0^{2\pi} e^{z\cos\theta}\,d\theta \tag{8}$$

Equation (7) will be recognized as a Rice distribution which is often seen representing the distribution of the envelope of a constant amplitude sinewave plus additive gaussian noise.

If r is interpreted as the voltage V which might be observed or measured at a particular location l, equation (7) directly gives the pdf for the voltage at this location, such that

$$p(V) = \frac{V}{\sigma^2}\exp\left(\frac{-(A^2 + V^2)}{2\sigma^2}\right) I_0\left(\frac{VA}{\sigma^2}\right) \tag{9}$$

In most cases, the predicted groundwave signal will be much larger in amplitude than the sum of the error signals. For this case, the pdf given by (9) is essentially gaussian with mean value approximately given by the predicted (expected) value A. Some examples of Rice distributions for a normalized value of $A = 1$, and various σ are shown in Figure 1.

3.0 VARIANCE OF MEASUREMENT DATA

With the pdf model for the voltage at a particular location given by equation (9), it is only necessary to specify A, the predicted voltage, and σ^2, the variance, to completely describe the pdf. As noted earlier, the variance can be based on analysis of measurement data using techniques described in [1]. The variance itself could be calculated as a single number representing all the measurement data on a radial, or be broken into several individual variances which apply to particular sections along the radial. These sections may be identified in terms of the amount of manmade construction, the terrain roughness, or other factors.

In general, the variance, σ^2, of a distribution of measured data points described by random variable X, is

$$\sigma^2 = E[X^2] - E[X]^2 \tag{10}$$

However, in this case we are interested in the distribution of the voltage *difference* between the measurement and the predicted values given by the voltage on the section of the groundwave propagation curve. This section of curve represents the expected, or mean, value. If we seek the distribution of the error, the mean value of the error is zero since it is equal to this predicted voltage value. Equation (10) then reduces to

$$\sigma^2 = E[X^2] \tag{11}$$

For a discrete set of N data points,

$$E[X^2] = \sum_{i=1}^{N} x_i^2\,P(x_i) \tag{12}$$

The $P(x_i)$ are the probabilities associated with a certain measured point. In the context of measurement points along a radial, the probability or weight as-

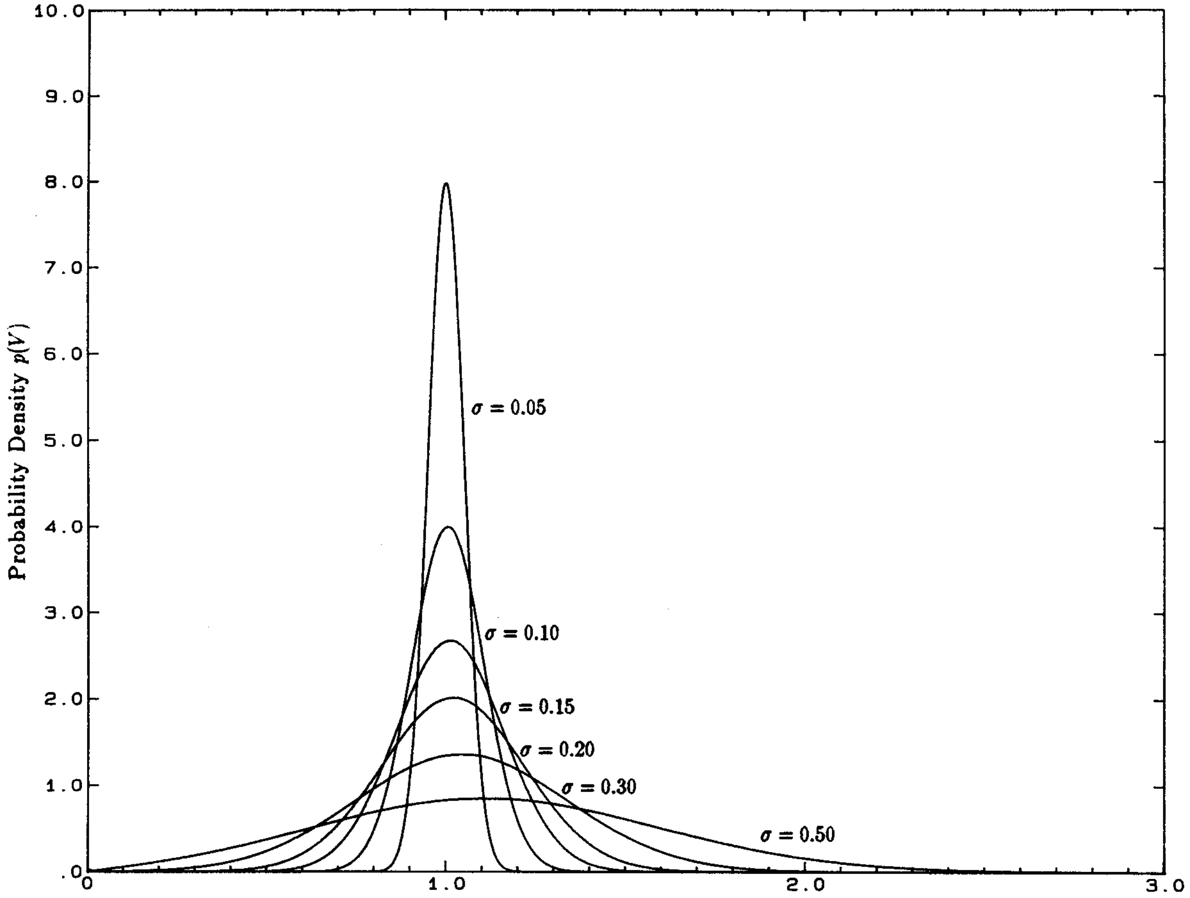

Fig. 1. Probability Density Function of Groundwave Signal Amplitude. $A = 1.0$

sociated with a measurement is proportional to the distance for which that measurement applies. That is, if $d(x_i)$ is the distance along a radial to point x_i, then

$$P(x_i) = \frac{(d(x_i) - d(x_{i-1}))/2 + (d(x_{i+1}) - d(x_i))/2}{d(x_N) + (d(x_N) - d(x_{N-1}))/2} \tag{13}$$

Where the total distance over which the data could be regarded as applicable is

$$d_t = d(x_N) + (d(x_N) - d(x_{N-1}))/2 \tag{14}$$

Using these $P(x_i)$ in equation (12), σ^2 can be found for any set of measurement points x_i. Using this σ^2 in equation (9), the pdf for the voltage is completely specified.

4.0 CALCULATED VARIANCE FROM MEASURED DATA

The formulas in Section 3.0 have been applied to several radials of measurement data analyzed using the techniques in [1]. The overall standard deviation σ of the measurement values around their expected values was about 15 percent, resulting in a variance of about 0.0225.

The variance seemed to be a function not only of the degree of manmade construction in the vicinity of the measurement locations, but also proportional to the distance from the transmitter (i.e. lower overall voltage measurements tended to be more scattered). Given the limited amount of data analyzed, and no detailed description of the measurement environment, it isn't possible to construct any particularly meaningful correlation between the calculated variance and other factors. With more data for analysis, and better descriptions of the measurement environments, it may be possible to eventually predict the variance which can be associated with a particular type of measurement location.

5.0 PROBABILITY OF THE SIGNAL STRENGTH EXCEEDING A GIVEN VALUE

With the pdf for the signal strength completely defined, the probability of the signal strength exceeding a given value z at a location can be found. If the predicted value A is known, and the variance is known, the probability of z being exceeded is simply the area under the curve described by (9) for all values $V > z$. For a Rice pdf, this value is given by the Q function. In general

$$Q(a, b) = \int_b^\infty \exp\left(\frac{-(a^2 + x^2)}{2}\right) I_0(ax)\, x\, dx \tag{15}$$

For this case, $a = A/\sigma$ and $b = z/\sigma$ so

$$P(V > z) = Q(A/\sigma, z/\sigma) \qquad (16)$$

If the value z is set to a median point, a location statistic can be generated representing the signal level exceeded at 50 percent of the locations, 100 percent of the time (assuming no time-dependent variations). For situations where $A >> \sigma^2$, the median value is essentially the predicted value, and the Rice distribution is essentially gaussian with mean value A.

Similarly, 10 percent and 90 percent location statistics can be calculated which could prove useful in developing more refined descriptions of groundwave coverage and interference contours for AM broadcast stations.

6.0 CONCLUSIONS

A derivation for the probability density function for AM broadcast groundwave signal voltages has been presented. With the assumption that the predicted signal voltage is time-invariant, and that it is modified by the addition of an error voltage representing the combined effects of reradiation from manmade objects and incompletely specified propagation path parameters, the pdf of the sum was shown to be a Rice-distributed random variable with mean essentially equal to the predicted voltage value and variance equal to the variance of the errant voltage contribution.

Preliminary analysis of actual field strength measurement data shows that a standard deviation of about 15 percent is calculated when considering all data taken from a wide variety of measurement environments.

The probability formulations presented here could be useful in producing more refined descriptions of AM broadcast groundwave coverage and interference contours. They can also be useful in calculating more accurate estimates of the probability of reception of audio or data signals transmitted on AM broadcast stations.

REFERENCES

[1] H. R. Anderson, "Systematic Bivariate Analysis of AM Field Strength Measurement Data," *IEEE Transactions on Broadcasting*, vol. BC-32, Number 2, 1986.